Int. J. Heat Mass Transfer. Vol. 12, pp. 233–237. Pergamon Press 1969. Printed in Great Britain

SHORTER COMMUNICATIONS

CONDENSATION OF A VAPOUR IN THE PRESENCE OF A NON-CONDENSING GAS

J. W. ROSE
Department of Mechanical Engineering, Queen Mary College, University of London, London

(*Received* 10 *April* 1968 *and in revised form* 3 *September* 1968)

NOMENCLATURE

D,	diffusion coefficient;
g,	gravitational acceleration;
h_{fg},	specific latent heat of vaporization;
M,	molecular weight;
q,	heat flux;
q_{Nu},	heat flux given by simple Nusselt theory in the absence of a non-condensing gas;
Sc,	ν/D;
Sp,	$(T_0 - T_w)\, k_f/h_{fg}\,\mu_f$;
T,	temperature;
t,	y/δ;
u,	x-component of velocity;
$\tilde{u}$,	a function of x having dimensions of velocity and defined in equation (4);
u_0,	longitudinal velocity of liquid–vapour interface;
v,	y-component of velocity;
W,	non-condensing gas concentration;
w,	$W - W_\infty$;
w_0,	$W_0 - W_\infty$;
X,	$(M_g - M_v)/\{M_g - W_\infty(M_g - M_v)\}$;
x,	distance along liquid–vapour interface from leading edge;
y,	distance normally from liquid–vapour interface.

Greek symbols

δ,	thickness of gas–vapour boundary layer;
μ,	absolute viscosity;
ν,	kinematic viscosity;
ρ,	density.

Subscripts

f,	condensate;
g,	gas (i.e. non-condensing component of mixture);
v,	vapour (i.e. condensing component of mixture);
w,	wall-condensate interface;
0,	condensate–vapour interface;
∞,	in gas–vapour mixture remote from the interface.

INTRODUCTION

THE PRESENT note concerns the problem of condensation from a gas-vapour mixture on a plane vertical surface, in the absence of forced convection. For the above case Sparrow and co-workers [1–3] have obtained boundary layer solutions for a variety of circumstances. The effect of free convection in the gas-vapour mixture, formerly omitted by Sparrow and Eckert [1], was included in the constant property* solution given by Sparrow and Lin [2]. In this work the effects of temperature on the mixture density and of heat-transfer in the gas-vapour mixture were not considered. More recently, a detailed treatment has been given by Minkowycz and Sparrow [3] in which the above restrictions have been lifted and, in addition, the role of temperature in the diffusion equation and that of diffusion in the energy equation, as well as the temperature discontinuity at the liquid-vapour interface have been considered.

Despite the fact that solutions may be found along the lines indicated [2, 3], a significant difficulty remains in that the necessary numerical solutions require extensive computation. For their solutions for steam-air mixtures, Minkowycz and Sparrow [3] remark that "even with a computer such as the CDC 1604, the time requirement was measurable in tens of hours". Even in the simpler case considered by Sparrow and Lin [2], separate solutions are required for each gas–vapour combination, remote gas concentration and condensation rate as well as for different temperature levels insofar as the temperature affects the mean properties to be adopted.

The approximate solution given here provides an algebraic equation relating the heat- and mass-transfer parameters and the relevant fluid properties. The case considered is that solved by Sparrow and Lin [2]. Minkowycz and Sparrow [3] have demonstrated that the considerations formerly included [2] are the dominating factors and that

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* With the usual exception of density in the bouyancy term of the momentum equation.

the additional effects included in [3] are generally less significant. Moreover, the close agreement between the results of Sparrow and Eckert [1] and Minkowycz and Sparrow [3], for condensation of pure superheated steam, where free convection was omitted in the former and included in the latter analysis, indicates that the influence of temperature on free convection, when a non-condensing gas is present, is of secondary importance.

ANALYSIS

With the co-ordinate system represented in Fig. 1, the equations expressing the conservation of mass, momentum and species in the boundary layer are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\left(1 - \frac{\rho_\infty}{\rho}\right) + \nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$u\frac{\partial W}{\partial x} + v\frac{\partial W}{\partial y} = D\frac{\partial^2 W}{\partial y^2} \qquad (3)$$

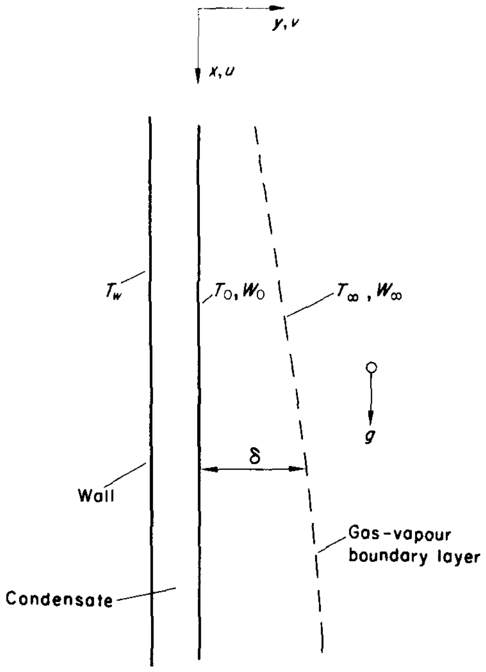

FIG. 1. Coordinate system.

where the properties of the mixture have been taken to be constant, with the exception of density in the buoyancy term in equation (2).

Suitable velocity and concentration profiles are:

$$u = u_0\left(1 - \frac{y}{\delta}\right)^2 + \tilde{u}\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2 \qquad (4)$$

$$\frac{W - W_\infty}{W_0 - W_\infty} = \left(1 - \frac{y}{\delta}\right)^2 \qquad (5)$$

where the two boundary layers are taken to have equal thicknesses*.

Then, with the boundary conditions that are satisfied by the above profiles and using the condition that the interface is impermeable to the non-condensing gas, the integral equations are:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\delta\int_0^1 u^2\,\mathrm{d}t\right) - v_0u_0 + \frac{\nu}{\delta}\left(\frac{\partial u}{\partial t}\right)_0 - gX\delta\int_0^1 w\,\mathrm{d}t = 0 \qquad (6)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\delta\int_0^1 uw\,\mathrm{d}t\right) + \frac{D}{\delta}\frac{W_\infty}{W_0}\left(\frac{\partial w}{\partial t}\right)_0 = 0 \qquad (7)$$

where:

$$t = y/\delta$$

$$w = W - W_\infty$$

$$X = \frac{M_g - M_v}{M_g - W_\infty(M_g - M_v)}$$

The solution of equations (6) and (7) using the assumed profiles [equations (4) and (5)] gives:

$$10\,Sp\,Sc\left(\frac{\mu_f\rho_f}{\mu\rho}\right)\left(\frac{W_\infty}{w_0}\right)^2\left(\frac{20}{21} + \frac{W_0}{W_\infty}Sc\right) + \frac{8}{Sp^2Sc}\left(\frac{\mu\rho}{\mu_f\rho_f}\right)\left(\frac{w_0}{W^0}\right)^2\left(\frac{5}{28}Sp - \frac{Xw_0}{3}\right) = \frac{100}{21}\frac{W_\infty}{W_0} - 2\frac{w_0}{W_0} + 8\,Sc \qquad (8)$$

where:

$$Sp = \frac{(T_0 - T_w)\,k_f}{h_{fg}\,\mu_f}$$

$$w_0 = W_0 - W_\infty\ .$$

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* This procedure has given satisfactory results in earlier cases [4–6]. In the case of ordinary free convection [4, 5], heat-transfer results in good agreement with the exact solutions were found, even for Prandtl number far from unity. In the present case, where the Schmidt number can never be far from unity, we might expect this assumption to be valid.

In obtaining equation (8), the values of the local condensation rate and interface velocity are taken to be those required by the Nusselt film theory.

COMPARISON WITH EXACT SOLUTIONS

Sparrow and Lin [2] obtained numerical solutions for two values of the Schmidt number, three values of the parameter $\rho_f \mu_f / \rho \mu$ and for five remote gas concentrations. The chosen values correspond roughly to steam–air mixtures for a temperature range 120 to 212°F. In Fig. 2, numerical solutions for $\rho_f \mu_f / \rho \mu = 150^2$ are compared with those given by equation (8). To avoid confusion, only the extreme values of the remote gas concentrations are given. The interfacial gas concentration is plotted against the non-dimensional temperature drop across the condensate layer, $(T_0 - T_w) k_f / (h_{fg} \mu_f)$. It may be seen that the approximate solution, while giving about the right dependence on *Sc*, overestimates the interfacial gas concentration. Similar results were obtained for the other two values of $\rho_f \mu_f / \rho \mu$ used by Sparrow and Lin [2].

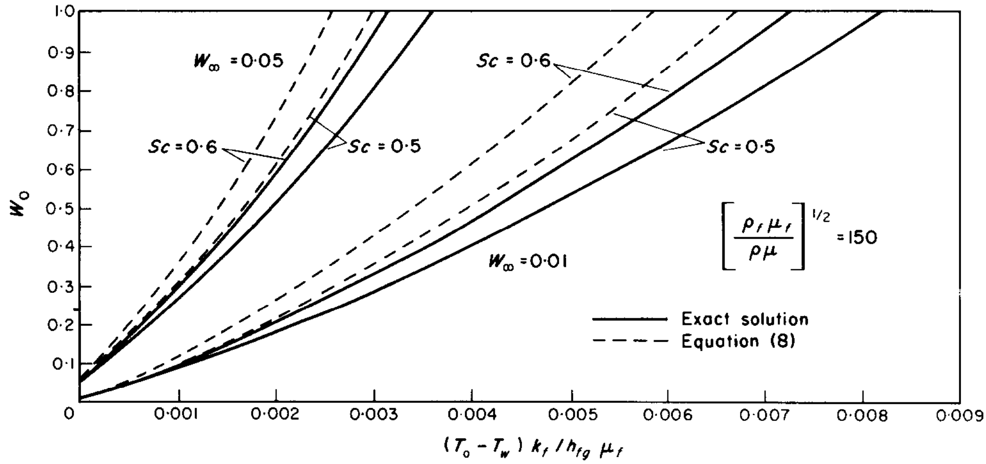

FIG. 2. Comparison of equation (8) with the exact constant property solution [2].

Since both the exact [2] and the present constant property solutions do not include an interface saturation (or a temperature jump) condition, the interfacial gas concentration is allowed to attain a value of unity. It may be the case, however, that the constant density approximation becomes invalid before this limit is reached since, as the interfacial gas concentration approaches unity, we have the situation where the theory, based on a constant density approximation, predicts a minimum-to-maximum density ratio in the gas–vapour mixture of about 0·63, for the range of remote gas concentrations considered above. If we restrict our attention to those cases for which $1 > \rho_\infty / \rho_0 > 0{\cdot}9$, i.e. for interfacial gas concentrations less than about 0·3, the error in the interfacial gas concentration given by equation (8) varies between zero and a maximum of about 0·05.

In Fig. 3 exact solutions of the variable property equations for steam–air mixtures given by Minkowycz and Sparrow [3] are compared with those given by equation (8). Both the curves of Minkowycz and Sparrow and those of the present work relate to bulk saturation conditions.

In obtaining the curves based on equation (8), the viscosity and density of the mixture were taken as the arithmetic means of their values at the interface and in the bulk. These latter were obtained having regard to both composition and temperature. The mixture viscosities were found by the method of Wilke [7]. The diffusion coefficient was evaluated at the arithmetic mean of the bulk and interface temperatures. For this purpose the method of Slattery and Bird [8] was adopted together with a value, for the diffusion coefficient, of 0·256 cm²/s at a temperature of 298°K and a pressure of 1 atm [9]. The condensate properties were evaluated at the reference temperature given by Minkowycz and Sparrow [3] and the latent heat at the interface temperature.

It may be seen from Fig. 3 that the results given by equation (8) follow the same general trends as those given by the exact solutions. At the lower gas concentrations equation (8) underestimates the heat transfer while at the higher concentrations the exact and approximate solutions virtually coincide. This was also the case for the other four temperature levels for which solutions were obtained by Minkowycz and Sparrow [3].

On the basis of the above comparisons it is considered that the approximate result should prove satisfactory in many practical circumstances where precise results are not required. Moreover, the dependence of the predictions of both the exact and approximate solutions, on the values of

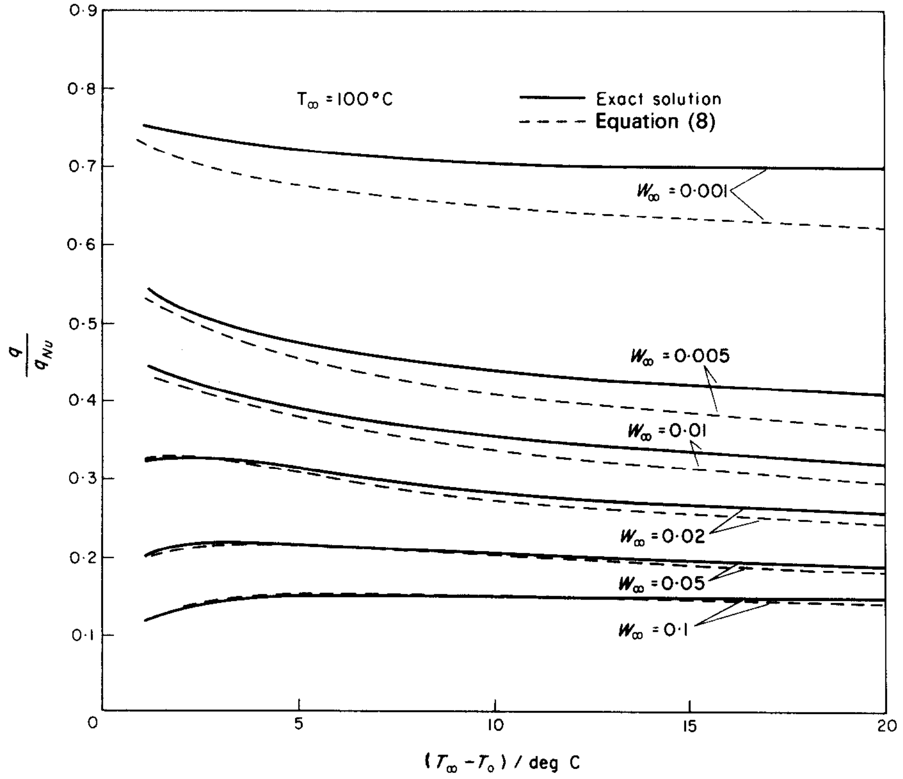

FIG. 3. Fractional reduction in heat transfer. Comparison of equation (8) with exact variable property solution [3].

the Schmidt number and the parameter $\rho_f\mu_f/\rho\mu$, indicates that the approximate method would be adequate in cases where the mixture properties were not well known. Also the fair agreement between the approximate solution and the exact variable property solution, over a wide range of the relevant variables, for steam–air mixtures, helps to confirm that the approximate equation should provide a generally satisfactory basis for many practical calculations. Even in those cases where precise results might be thought necessary, the approximate solution should prove useful in preliminary calculations.

It is possible that other methods of estimating the average properties of the gas–vapour mixture, appropriate for use in equation (8) might lead to closer agreement with the exact variable property solution. However, in view of the generally satisfactory agreement found when using the present estimates, and of the absence of exact solutions for other gas–vapour mixtures, this was not considered to be worth pursuing at present.

Finally, it should be noted that neither exact nor approximate solution is valid for the case where the molecular weight of the non-condensing gas is smaller than that of the vapour. Furthermore, we should not expect either result to hold for Grashof numbers below those necessary to the validity of the boundary layer approximation nor above those at which turbulence occurs.

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Int. J. Heat Mass Transfer. Vol. 12, pp. 237–239. Pergamon Press 1969. Printed in Great Britain

NUSSELT CONDENSATION OF *n*-BUTYL ALCOHOL

L. SLEGERS[*] and R. A. SEBAN[†]

Department of Mechanical Engineering, University of California, Berkeley, U.S.A.

(*Received* 27 *February* 1968 *and in revised form* 31 *May* 1968)

NOMENCLATURE

f, condensation coefficient;
g, acceleration of gravity;
h_{fg}, latent heat of vaporization;
h_{if}, interfacial heat transfer coefficient;
k, thermal conductivity;
L, height of condensing surface;
P, pressure
R, universal gas constant;
q, heat flux;
T, temperature;
u, velocity in liquid film;
V, specific volume;
y, coordinate normal to condensation surface;
δ, liquid film thickness;
ρ, density;
μ, dynamic viscosity.

Subscripts

s, liquid surface at liquid-vapor interface;
v, vapor;
w, wall;
csate, condensate measurement;
cond, conduction measurement;
Nu, Nusselt solution.

INTRODUCTION

Heat flux measurements were made of the laminar film condensation of *n*-butyl alcohol on a vertical surface. The measurements were compared with the Nusselt solution in an attempt to ascribe any difference between predicted and measured heat flux values to interfacial resistance, and thus to obtain a value for the condensation coefficient. The lowest vapor pressure obtainable in the present experiment was 9·4 mmHg (85 degF). At this pressure and above, the results indicate that the condensation coefficient is larger than 0·25. This is substantially larger than values reported for this class of liquids by previous experimenters. Kicska and Smith [1] obtained a most probable value for *sec*-butyl alcohol of 0·025. Measurements performed on methanol by Delaney [2] have yielded values of 0·017 at 7 degC and 0·030 at −27 degC while similarly low values are quoted in [2] from other literature.

DISCUSSION

The apparatus was a rebuilt version of that described by Mills [3], in which the condensation surface was the vertical face of a thick copper block 2-in. wide by 5-in. high, and was cooled on the opposite face with a refrigerated coolant. The lateral faces of the block were insulated by the nature of the block containment. In the block were embedded two columns of six thermocouples each; the columns being located at 1 and 2 in. from the condensing surface. These temperature measurements enabled a heat flux determination and the surface temperature was obtained by forward extrapolation. A further heat flux measurement was made by collecting condensate. Vapor temperature was measured with a thermocouple, and verified with a pressure measurement through available saturation pressure information [6]. Saturation conditions were at all times maintained.

Careful precaution was taken against the presence of noncondensable gases in the vapor. The apparatus was assembled using high vacuum components and soldered connections only. Upon assembly extensive tests for leaks

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[*] Research Assistant.

[†] Professor of Mechanical Engineering.